Warning: the horizon is closer than you think.

A bit of Reality by Phillip Berrie

I was reading a chapter from a story set on Mars when the author had their characters use a telescope to see an object lying on the ground fifteen kilometres away from their position on a flat plain. At the time this triggered my nerd sense as I was very much aware that the horizon is normally a lot closer than people think and this must especially be the case on Mars where the radius of curvature of the planet is much more extreme than Earth.

So I hit the web and did some research and determined that those characters certainly couldn't have seen this object because it was well and truly below the horizon for them. I was also taken aback at just how the numbers came out so, as speculative fiction writers tend to set their stories on other worlds a lot of the time, I thought the following information, or at least a heads up about this problem, might be of value to other writers.

So, you have been warned and ... here comes the maths.

The Maths:

The distance to the horizon on a sphere (d), an assumption which should approximate most planets, is based on the radius of the sphere (R) and the observer's height above the surface (h). The best equation I found, which can even be used for satellites, is as follows:

$$d = \sqrt{2Rh + h^2}$$

It is not simple to convert this formula to one that can generally calculate the height required to see a certain distance. However, where relatively small values of h are involved, the h-squared term becomes irrelevant so an approximate value can be obtained using the following formula:

$$h = \frac{d^2}{2R}$$

Note: For larger heights you could do what I did and use a spreadsheet with the first formula and work backwards. Its awkward, but once you've got the formula right it becomes very easy to determine an approximate height for your distance.

And now for some numbers: For the case of Mars, which has a radius of approximately 3,400 kms, an observer 2 metres above the surface of a flat plain can see approximate 3.7 kilometres. On Earth they could see a little over 5 kilometres. In the Martian example, the observer would need to be elevated to a height of 35 metres to see an object lying on the ground 15 kilometres away. On Earth, this would be about 20 metres.

References:

For more than you probably ever want to know about the distance to the horizon have a look at the following Wikipedia reference.

http://en.wikipedia.org/wiki/Horizon

And this site has a slightly different explanation and even includes a distance calculator.

http://newton.ex.ac.uk/research/qsystems/people/sque/physics/horizon/

I really liked the first comment on this last page. Sounds to me like a lot of authors have already been here.

N.B. Please note that I although I use the Wikipedia (and WikiMedia Commons) for a lot for references, this is for expediency and the familiarity of my readers. Anyone interested in further studies should make use of the references where available and understand that the Wikipedia is a co-operative project contributable to by anyone and must always be looked at in that light.

Was this of use to you?
See more Reality Bit(e)s at
www.phillberrie.com.au/Reality